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A NUMERICAL-ANALYTIC INVESTIGATION OF THE FLUTTER OF A PLATE OF ARBITRARY SHAPE IN THE PLANE†

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Using a new formulation of the flutter problem [1] and earlier results [2, 3], an algorithm is proposed for investigating the flutter of a plate of arbitrary shape in the plane for arbitrary homogeneous boundary conditions. © 1997 Elsevier Science Ltd. All rights reserved.

The majority of investigations of panel flutter apply to rectangular plates in the special formulation when the velocity vector of the flow is parallel to one of the sides (see the review in [4]). The flutter of circular and elliptic plates has been studied by the Bubnov–Galerkin method in the two-term approximation [5] (in the latter case the velocity vector is parallel to the major axis of the ellipse). Thus, a large class of important practical problems has not been covered. Below we fill this gap.

1. FORMULATION OF THE PROBLEM AND DISCRETIZATION

An investigation of the stability of vibrations of a thin plate of arbitrary shape occupying a domain G with boundary ∂G in the x, y plane with gas flowing past the plate leads to the eigenvalue problem [1] for the deflection amplitude $\varphi = \varphi(x, y)$, $(x, y) \in G$

$$D\Delta^2\varphi - \beta \mathbf{V}\,\mathrm{qrad}\,\varphi = \lambda\varphi; \quad D = Eh^3/(12(1-\nu^2)), \quad \beta = kp_0/c_0 \tag{1.1}$$

$$\varphi|_{\partial G} = 0, \quad M\varphi|_{\partial G} = 0 \tag{1.2}$$

Here E and ν are Young modulus and the Poisson's ratio of the material of the plate, h is the plate thickness, $\mathbf{V} = (V_x, V_y)$ is the gas velocity vector, p_0 and c_0 are the pressure and velocity of sound in the unperturbed flow, and k is the polytrope index of the gas.

The eigenvalue λ is related to the oscillation frequency by the equation

$$\lambda = -\rho h\omega^2 - \beta\omega \tag{1.3}$$

where ρ is the density of the plate material.

In (1.2) M is a differential operator known in the theory of plates, defined by the type of boundary conditions. A method of solving eigenvalue problem (1.1)–(1.3) has been described for an arbitrary operator M.

The plate vibrations will be steady or not, depending on whether Re $\omega < 0$ or Re $\omega > 0$. If $\lambda_1 = \alpha_1 + \beta_1 i$ is the eigenvalue of least modulus, then, by (1.3), $F(\alpha_1, \beta_1) > 0$ or $F(\alpha_1, \beta_1) < 0$ satisfy the given inequalities, where $F(\alpha_1, \beta_1) = \alpha_1\beta^2) - \rho h\beta^2{}_1$. Since $\alpha_1 = \alpha_1(V)$, $\beta_1 = \beta_1(V)$, it follows that the equation $F(\alpha_1, \beta_1) = 0$ defines a neutral curve and the corresponding rate of flutter. We are therefore concerned with finding the zeros of $F(\alpha_1(V), \beta_1(V)$ given the flow velocity vector.

We denote by l the characteristic dimension of G and introduce the following dimensionless coordinates and parameters (denoted by primes)

$$x = x'l, \quad y = y'l, \quad E = E'p_0, \quad h = h'l$$

$$\rho = \rho' p_0/c_0^2, \quad \omega = \omega' c_0/l, \quad V = V'c_0, \quad \varphi = \varphi' l$$

Substituting into (1.1) and (1.3), we can verify that the system in the dimensionless variables preserves its form if β is replaced by a dimensionless parameter k. The prime will henceforth be omitted.

In place of the Cartesian coordinates x, y we introduce curvilinear coordinates r, ϑ by $x = u(r, \vartheta)$, $y = v(r, \vartheta)$. If the Cauchy–Riemann conditions

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$$\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \vartheta},\quad \frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \vartheta}$$

are satisfied, then the system of coordinates r, ϑ is orthogonal. Now we introduce functions $u(r, \vartheta)$ and $v(r, \vartheta)$ such that

$$\psi(\zeta)=u(r,\vartheta)+iv(r,\vartheta),\quad \zeta=r\exp(i\vartheta)$$

defines a conformal mapping of the circle $|\zeta|=r\leq 1$ onto G. Then (1.1) takes the form

$$D\Delta(|\psi'(\zeta)|^{-2}\Delta\varphi)-k((V_x u_r+V_y v_r)\frac{\partial\varphi}{\partial r}+\frac{1}{r}(V_y u_r-v_r V_x)\frac{\partial\varphi}{\partial\vartheta})=\lambda|\psi'(\zeta)|^2\varphi \tag{1.4}$$

$$\left(u_r=\mathrm{Re}\left(\frac{\psi'(\zeta)\zeta}{r}\right),\quad v_r=\mathrm{Im}\left(\frac{\psi'(\zeta)\zeta}{r}\right)\right)$$

in the coordinates (r, ϑ), the boundary conditions (1.2) being transformed in the well-known way [3].

Henceforth we shall assume G to be a simply connected domain and ∂G to be a Lyapunov curve. This ensures that the fundamental Riemann theorem and the theorem on the correspondence of boundaries are satisfied [6].

We put

$$f(r,\vartheta)=\Phi(r,\vartheta)+\lambda|\psi'(\zeta)|^2\varphi$$

$$\Phi(r,\vartheta)=k\left[\left(V_x u_r+V_y v_r\right)\frac{\partial\varphi}{\partial r}+\left(V_y u_r-V_x v_r\right)\frac{\partial\varphi}{r\partial\vartheta}\right]$$

and write (1.4) as

$$D\Delta(|\psi'(\zeta)|^{-2}\Delta\varphi)=\Phi(r,\varphi)+\lambda|\psi'(\zeta)|^2\varphi \tag{1.5}$$

It is now obvious that the discretization of boundary-value problem (1.5), (1.2) is completely analogous to that described earlier [3] for a biharmonic operator.

In the circle we choose a grid consisting of m circles whose radii are the positive roots of the Chebyshev polynomial T_{2m} and in each circle we introduce a uniform grid consisting of $N = 2n + 1$ points, denoting the grid nodes by ζ_j. Here and henceforth $i, j = 1, 2, \ldots, S$, where $S = mN$. We change [3] to an integral equation and apply to $f(r, \vartheta)$ Babenko's interpolation formula for functions of two variables in a circle [2]. As a result, we obtain

$$\varphi_i-\frac{1}{D}\sum_j B_{ij}z_j^{-1}\Phi_j=\frac{\lambda}{D}\sum_j B_{ij}\varphi_j,\quad z_j=|\psi'(\zeta_j)|^2 \tag{1.6}$$

Here B is the matrix of the discrete operator inverse to the biharmonic operator with the given boundary conditions.

We denote by $D^{(r)}$ and $D^{(\vartheta)}$ the matrices of derivatives with respect to r and ϑ obtained by differentiating the interpolation formula. Then

$$\Phi_j=a_j\sum_l D_{jl}^{(r)}\varphi_l+r^{-1}b_j\sum_l D_{jl}^{(\vartheta)}\varphi_l$$

$$a_j=k(V_x u_r+V_y v_r)|_{\zeta=\zeta_j},\quad b_j=k(V_y u_r-V_x v_r)|_{\zeta=\zeta_j}$$

Now, from (1.6) we obtain the following eigenvalue problem in matrix form for the vector φ of length S whose components are the values of the corresponding function at the grid nodes

$$\varphi=\lambda D^{-1}A^{-1}B\varphi$$

$$A=I-D^{-1}BZ^{-1}(aD^{(r)}+bD^{(\vartheta)}),\quad Z^{-1}=\mathrm{diag}(z_1^{-1},\ldots,z_s^{-1})$$

$$a=\mathrm{diag}(a_1,\ldots,a_s),\quad b=\mathrm{diag}(b_1,\ldots,b_s)$$

Let μ be the eigenvalue of $D^{-1}A^{-1}B$ with the largest modulus. Then $\lambda = 1/\mu$ will be the desired eigenvalue with the least modulus.

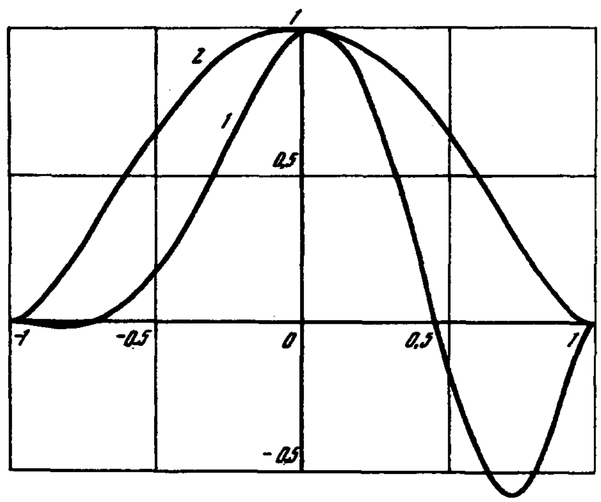

Fig. 1.

2. DISCUSSION OF THE METHOD

The method described for solving system (1.1)–(1.3) is based on Babenko's global interpolation formula for functions of two variables in a circle. The properties of this interpolation are such that it reacts to the degree of smoothness of the interpolated function: a smoother function will be approximated more accurately by the interpolation formula, so the proposed algorithm turns out to be more accurate.

Since the degree of smoothness of the solution is unknown in advance (the current state of the problem is presented in [7]), it is difficult in practice to use error estimates when computing the eigenvalues [8]. However, a qualitative argument can be put forward for the reliability of the algorithm: (a) interpolation by polynomials (algebraic and trigonometric ones) is used; polynomials are known [9] to approximate smoother functions with higher accuracy; (b) the grid nodes along the radius have been chosen to be the roots of the Chebyshev polynomial T_{2m}, which accumulate near the boundary the more rapidly the larger the value of m. The Lebesgue constant is thus minimized and the second boundary condition is satisfied to a greater degree, which is important for equations with a small parameter of the leading operator. A final argument supporting the reliability of the algorithm can be made on the basis of numerical experiments only.

3. RESULTS

All the computations were performed for the following parameter values: $k = 1.4$, $\nu = 0.33$, $c_0 = 331.26$ m/s, $p = 1.0133 \times 10^5$ Pa, $E = 6.867 \times 10^{10}$ Pa, $\rho = 2.7 \times 10^3$ kg/m^3, and a relative plate thickness $h = 3 \times 10^{-3}$. The operator M in (1.2) was taken to be the normal derivative (rigid attachment): a) the circle $r \leq 1$, was taken as G and $\psi(\zeta) = \zeta$, $V_x = -V$, $V_y = 0$.

For the dimensionless critical velocity we obtained $V^* = 0.2798$. The graphs in Fig. 1 enable us to form an opinion about the shape of deflections: we present Re $\varphi(x, 0)/\varphi_{\max}$ (curve 1) and Re$\varphi(0, y)/\varphi_{\max}$ (curve 2); b) G bounded by an epitrochoid $\psi(\zeta) = \zeta(1 + 0.1\zeta^4)$, $V_x = -V$, $V_y = 0$.

The critical velocity if $V^* = 0.2798$. The graphs characterizing the shape of the deflections are practically the same as the corresponding curves presented in the figure for the case of a circle. When $V_x = V_y = -V/\sqrt{2}$ we obtained $V^* = 0.2789$ for the critical velocity. The graphs of Re $\varphi(x, x)$ and Re $\varphi(x, -x)$ do not contain any new information compared with those presented.

All computations were performed using three grids: 9×31, 11×31, 13×31 (the number of nodes along the radius and around the circle, respectively). In all cases the results agree to within 10^{-4}, showing that the proposed method is effective, economical and reliable.

The results obtained are also of direct interest: the problem of the flutter of a round plate has not been studied sufficiently, nor has the solution of the second problem, solved above, been considered before.

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